Asymmetrical two-atom entanglement in a coated microsphere

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Abstract

We study evolution of entanglement of two two-level atoms placed inside a multilayered microsphere. We show that due to inhomogeneity of the field modes this entanglement essentially depends on the atomic positions (asymmetrical entanglement) and also on the detuning between the atomic transitions and field frequencies. The robust and complete entanglement can be achieved even in the resonant case when the atoms have different effective coupling constants, and it can be extended in time if the detuning is large enough. We study analytically the lossless case and estimate numerically the effect of dissipative processes.

I. INTRODUCTION

Recently, essential progress in fabrication and determination of optical properties of different kinds of microcavities with sizes about $0.1 - 20\mu m$ which contain semiconductor nanoclusters or quantum dots (QDs) has been achieved [see [1], [2] and references therein]. When semiconductor QDs are embedded in a spherical microcavity, the QD luminescence can be coupled with eigen modes of the electromagnetic field of the microcavity and a lower threshold of stimulated emission (or lasing modes) of QDs can be achieved. In recent papers [2],[3],[4] a coupling between the optical emission of embedded $CdSe_xS_{1-x}$, QDs and spherical cavity modes was studied and a strong whispering gallery mode (WGM) resonance with high Q factors is registered in the photoluminescence spectra. Recently [5] quantum-confined semiconductor nanorods were used as highly polarized nanoemitters for active control of the polarization state of microcavity photons.

Until now the modes with small numbers of spherical harmonics (SNM) are essentially less well-studied compared with whispering gallery modes (WGM) due to their rather low Q-factor caused by significant radiating losses. A possibility of strongly increasing the Q-factor of the microsphere was proposed in several papers, see e.g. [6], [7], [8]. The main idea consists in coating a microsphere by alternative layers of a spherical stack, which results in an increase of the Q factor up to values comparable for WGM, i.e. $10^7 - 10^9$.

In a system with small mean photon number two spatially separated atoms in a cavity become entangled at some time moments as a result of sharing the re-radiated photons [9], [10], [11], [12], [13]. One of these schemes has been realized using Rydberg atoms coupled one by one to a high Q microwave superconducting microcavity [14]. In inhomogeneous structures, like multilayered microspheres, the quantized field properties are quite different from the unbounded case because of a non-uniformity of the cavity field, which becomes important for the entanglement dynamics. In spite of numerous experimental obstacles, mainly related with the decoherence problem, it seems very natural to entangle atoms placed in high-Q cavities (like microspheres) via interaction with modes of the cavity quantized field. A simple scheme for the generation of two-atom maximally entangled states via dispersive interaction was proposed in [15]. A number of papers report studies of the evolution of the entanglement in an atomic subsystem resonantly interacting with a single mode of the cavity field (two-atom Tavis-Cummings model) [16], [17], [18]. A robust generation of many-particle

entanglement in various configurations has been discussed in [19], [20], [21]. Experimentally a robust entanglement was recently studied in[22] where an entanglement lasting for more than $20s$ was observed in a system of two trapped $Ca+$ ions. Authors [23] have shown that the degree of entanglement between the two atoms strongly depends on the mean photon number and the strength of two-photon correlations.

In [24] a scheme for entangling N two-level atoms located close to the surface of a dielectric microsphere and atoms resonantly interacting with the field was considered. It was shown that in the particular case of two atoms located at diametrically opposite positions a perfect entanglement cannot be achieved even in the strong-coupling regime.

In this paper we study two-atom entanglement interacting with field modes inside a microsphere covered with spherical dielectric alternating layers (coated microsphere). We are mainly interested: *(i)* in the frequency range of the high reflectivity field in $\lambda/4$ -stack; *(ii)* in the case when identical atoms are located asymmetrically inside the microsphere (i.e. the system is not symmetric with respect to a permutation of initially excited and non-excited atoms), so that the field inhomogeneity leads to different effective atom-field coupling constants ; and *(iii)* the atomic transitions can be both resonant and well detuned from the field peak frequency.

The paper is organized as follows. In Section II we discuss basic equations for two atoms placed into a coated microsphere and the solution for this case. In Section III we present an analytical solution for probability amplitudes and apply it to studying the atomic concurrence. In Section IV we present a numerical study of the concurrence (tangle) dynamics. In the last Section, we discuss and summarize our conclusions.

II. BASIC EQUATIONS

Consider two identical two-level atoms coupled to a quantized electromagnetic cavity field in a coated microsphere (Fig.1).

Let us assume that the atoms are sufficiently far from each other, so that the interatomic Coulomb interaction can be ignored. In this case, the electric dipole and rotating wave approximations can be applied and the Hamiltonian for the atom-cavity system ($\hbar = 1$) is

FIG. 1: Geometry of coated microsphere with two atoms.

given by [25], [26]

$$H = H_0 + H_1, \tag{1}$$

$$H_0 = \hat{H} = \int \mathrm{d}^3\mathbf{r} \int_0^\infty d\omega\, \hbar\omega\, \hat{f}^\dagger(\mathbf{r},\omega)\hat{f}(\mathbf{r},\omega) + \sum_{j=1,2} \frac{1}{2}\omega_j \widehat{s}_{jz},$$

$$H_1 = -\sum_j [\widehat{s}_j^\dagger \hat{E}^{(+)}(\mathbf{r}_j) d_j + H.c.],$$

where ω_j is the atomic transition frequency ($\omega_1 = \omega_2 = \omega_{at}$) , $s_{z,\pm j}$, $j = 1, 2$ are the atomic operators corresponding to the j-th atom and obeying standard $su(2)$ commutation relations, $[s_\pm, s_z] = \pm s_\pm, [s_+, s_-] = 2s_z$, d_j are atomic dipoles. Here $\hat{f}(r,\omega)$ and $\hat{f}^\dagger(r,\omega)$ are bosonic operators which play the role of the fundamental variables of the electromagnetic field and the medium, including a reservoir necessarily associated with losses in the medium. The electric-field operator is expressed in terms of $\hat{f}(r,\omega)$ as [25], [26],

$$\hat{E}^{(+)}(\mathbf{r}) = i\sqrt{\frac{\hbar}{\pi\varepsilon_0}} \int_0^\infty d\omega \frac{\omega^2}{c^2} \int \mathrm{d}^3\mathbf{r}' \sqrt{\varepsilon_{\mathrm{I}}(\mathbf{r}',\omega)}\mathbf{G}(\mathbf{r},\mathbf{r}',\omega)\hat{f}(\mathbf{r}',\omega), \tag{2}$$

with

$$\left[\hat{f}_i(\mathbf{r},\omega), \hat{f}_j^\dagger(\mathbf{r}',\omega')\right] = \delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'), \tag{3}$$
$$\left[\hat{f}_i(\mathbf{r},\omega), \hat{f}_j(\mathbf{r}',\omega')\right] = 0 = \left[\hat{f}_i^\dagger(\mathbf{r},\omega), \hat{f}_j^\dagger(\mathbf{r}',\omega')\right],$$

where $G(\ r,\ r',\omega)$ is the classical Green tensor satisfying the equation

$$\left[\frac{\omega^2}{c^2}\epsilon(\ \mathbf{r},\omega) - \boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\right]\mathbf{G}(\ \mathbf{r},\ \mathbf{r}',\omega) = -\delta(\ \mathbf{r}-\ \mathbf{r}') \tag{4}$$

together with the boundary condition at infinity [$\delta(\mathbf{r})$ is the dyadic δ-function]. Here $\varepsilon(\mathbf{r},\omega) = \varepsilon_{\mathrm{R}}(\mathbf{r},\omega) + i\varepsilon_{\mathrm{I}}(\mathbf{r}',\omega)$ is the complex dielectric permittivity. We look for the solution of the Schrödinger equation with the Hamiltonian (1) in a single excitation manifold in the form

$$|\Psi(t)\rangle = C_1(t)\,|0\rangle\,|e_1g_2\rangle + C_2(t)\,|0\rangle\,|g_1e_2\rangle + \tag{5}$$
$$+\,|g_1g_2\rangle\int \mathrm{d}^3\mathbf{r}\int_0^\infty d\omega\,[C_{3i}(\mathbf{r},\omega,t)|\{1_i(\mathbf{r},\omega)\}\rangle],$$

where $|e_k\rangle$ ($|g_k\rangle$) denotes the excited (ground) atomic state of k-th atom. Correspondingly, $|\{1_i(r,\omega)\}\rangle = \hat{f}_i^\dagger(r,\omega)|\{0\}\rangle$ is a single photon Fock state and, $|\{0\}\rangle$ is the vacuum state of the rest of the system. Note that this state is not a photonic state in general, but a state of the macroscopic medium dressed by the electromagnetic field [27],[28],[26],[24].

For simplicity we study the frequency range close to the microsphere resonance with the frequency ω_f, when the Green function can be written as

$$G(\mathbf{r},\mathbf{r}',\omega) = G(\mathbf{r},\mathbf{r}',\omega)\cdot\delta(\omega-\omega_f). \tag{6}$$

The effect of broadening of such a line due to dissipation is studied in Sec.IV numerically. For further references we show in Fig.2 the typical frequency spectrum of Green's function, calculated numerically. Let us assume that the atomic dipoles are parallel to the surface of the microsphere (similar to the situation considered in [5]), so only tangential components of Green's tensor (e.g. $\mathbf{G}\varphi\varphi$) give a contribution.

Projecting $|\Psi(t)\rangle$ in (5) onto $|0\rangle\,|e_ig_k\rangle$ and $|\{1_i(r,\omega)\}\rangle\,|g_1g_2\rangle$ states, we obtain the following equations for the probability amplitudes C_i:

$$\dot{C}_{1,2}(t) = -iB(a_{1,2},\omega_f,t), \tag{7}$$

$$\dot{B}(\mathbf{r},\omega_f,t) = i\Delta\omega B(\mathbf{r},\omega_f,t) - i\overline{\mathbf{G}}(\mathbf{r},a_1,\omega_f)\ C_1(t) - i\overline{\mathbf{G}}(\mathbf{r},a_2,\omega_f)\ C_2(t),$$

FIG. 2: (a), (b) and (c). Frequency spectrum of imaginary parts of tangential component of the dyadic Green's function $\mathrm{Im}(G_{\varphi\varphi}(r,r',f))$, $f=\omega/2\pi$ for 7-layered system (microsphere coated with 5 alternating $\lambda/4$ layers), with atomic positions $a_1=0.9\mu m$ and $a_2=1.1\mu m$. Refraction indexes of the layers are $n_4=1.5+i2\cdot 10^{-4}$ (glass, bottom microsphere, $1\mu m$), $n_3=3.58+i10^{-3}(Si,\ 0.12\mu m)$, $n_2=1.46+i3\cdot 10^{-3}$ (SiO_2, $0.3\mu m$) and $n_1=1$ (surrounding space). (a) $\mathrm{Im}(G_{\varphi\varphi}(a_1,a_1,f))$; (b) $\mathrm{Im}(G_{\varphi\varphi}(a_1,a_2,f))$; and (c) $\mathrm{Im}(G_{\varphi\varphi}(a_2,a_2,f))$; (d) radial dependence of $\mathrm{Im}(G_{\varphi\varphi}(r,a_2,f_f))$, where the atom is placed in $a_2=0.9\mu m$ and the field's peak frequency is $f_f\ =241.7THz$. Dash line in (d) shows the refraction indexes of the spherical stack structure.

where $\mathbf{r}$ is coordinate vector, $a_{1,2}$ are the positions of the atoms in the microsphere, $\Delta\omega=\omega_f-\omega_{at}$, and

$$B(\mathbf{r},\omega_f,t)=d_i\int \mathrm{d}^3\mathbf{r}'\cdot\alpha\,\mathbf{G}_{ik}(a,\mathbf{r}',\omega_f)C_{3k}(\mathbf{r}',\omega_f,t), \tag{8}$$

$$\overline{\mathbf{G}}(\mathbf{a},\mathbf{r}',\omega)=\kappa d_i d_k\,\mathrm{Im}(\mathbf{G}_{ik}(\mathbf{a},\mathbf{r}',\omega)), \tag{9}$$

where $\alpha=i\sqrt{\varepsilon_I(\mathbf{r},\omega_f)/\pi\varepsilon_0}\omega_f^2/c^2$ and $\kappa=\omega_f^2/c^2\pi\varepsilon_0$. Eliminating $B(\mathbf{r},\omega_f,t)$ from (7) we obtain after minor algebra closed equations for $C_{1,2}(t)$ in matrix form as follows

$$\frac{d^2\mathbf{q}}{dt^2}-i\Delta\omega\frac{d\mathbf{q}}{dt}+\mathbf{A}\cdot\mathbf{q}=0, \tag{10}$$

where

$$\mathbf{q} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} \overline{G}(1,1) & \overline{G}(1,2) \\ \overline{G}(2,1) & \overline{G}(2,2) \end{bmatrix}.$$

To derive (10) the identity [25] $\mathrm{Im}\, G_{kl}(\mathbf{r}, \mathbf{r\prime}, \omega) = \int \mathrm{d}^3\mathbf{s}\, (\omega^2/c^2)\, \varepsilon_{\mathrm{I}}(\mathbf{s}, \omega) G_{km}(\mathbf{r}, \mathbf{s}, \omega) G^*_{lm}(\mathbf{r}', \mathbf{s}, \omega)$ was taken into account. From now on we adopt the convention of summation over repeated vector-component indices.

The general solution of Eq.(10) has the form

$$C_k(t) = \sum_{j=1}^{4} c_{kj} e^{i\omega_j t},\ k = 1, 2, \tag{11}$$

where the frequencies ω_j are solutions of the eigenvalue problem $\det\left[(-\omega^2 + \omega\Delta\omega)\, \delta_{kl} + A_{kl}\right] = 0$ or

$$\left(-\omega^2 + \omega\Delta\omega\right)^2 + \left(-\omega^2 + \omega\Delta\omega\right) Tr\{\mathbf{A}\} + \det(\mathbf{A}) = 0. \tag{12}$$

Following [29] we rewrite the Green tensor for a multilayered microsphere as follows

$$\mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) = \mathbf{G}^V(\mathbf{r}, \mathbf{r}', \omega)\delta_{fs} + \mathbf{G}^{(fs)}(\mathbf{r}, \mathbf{r}', \omega), \tag{13}$$

where $G^{\mathrm{V}}(r, r', \omega)$ represents the contribution of the direct waves from the radiation sources in an unbounded medium, f and s denote the layers where the field point and source point are located, δ_{fs} is the Kronecker symbol, and the scattering Green tensor $\mathbf{G}^{(fs)}(\mathbf{r}, \mathbf{r}', \omega)$ describes the contribution of both multiple reflection and transmission. The Green tensor $\mathbf{G}^{(fs)}$ in general can be expanded as

$$\mathbf{G}^{(fe)}(\mathbf{r}, \mathbf{r}', \omega) = \frac{ik_s}{4\pi} \sum_{p=e,o} \sum_{n=1}^{\infty} \sum_{m=0}^{n} \frac{2n+1}{n(n+1)} \frac{(n-m)!}{(n+m)!} (2-\delta_{0m}) \mathbf{G}^{(f,e)}_{pnm}(\mathbf{r}, \mathbf{r}', \omega), \tag{14}$$

where $G^{(f,e)}_{cnm}(\mathbf{r}, \mathbf{r\prime}, \omega)$ is a particular Green tensor, n is the spherical and m is the azimuth quantum numbers of a microsphere, $k_i = \omega n_i/c$, $n_i = \sqrt{\varepsilon_i(\omega)}$ is a refraction index. General recurrent formulas and particular representations of the Green tensor $G^{(f,e)}_{pnm}(\mathbf{r}, \mathbf{r\prime}, \omega)$ can be found in Ref.[29].

Generally analysis of the Green tensor (14) requires intensive computation. In the simplest case when atoms are located at positions with the same value of the field amplitude, we have $\overline{G}(i,j) = \overline{G}$ and $\det(\mathbf{A}) = 0$. This case is symmetrical with respect to

permutation of the atoms and one can easily obtain the solution $\omega_1 = 0$, $\omega_2 = \Delta\omega$ and $\omega_{3,4} = (\Delta\omega/2) \pm \left[(\Delta\omega/2)^2 + 2\overline{G}\right]^{1/2}$. However, experimentally, a symmetric location of the atoms with respect to the center of the microsphere is difficult to achieve [13]. In the simplest nontrivial case we have to take into account the nonuniformity of the field. In this case we have $\overline{G}(1,1) \neq \overline{G}(2,2)$, but the coefficients $\overline{G}(i,k)$ can be written as follows

$$\overline{G}(1,1) = \chi_1^2, \;\; \overline{G}(2,2) = \chi_2^2, \, \overline{G}(1,2) = \overline{G}(2,1) = \chi_1\chi_2, \tag{15}$$

so that the condition

$$\overline{G}(1,1) \cdot \overline{G}(2,2) = \overline{G}(1,2)^2 \tag{16}$$

is fulfilled. We have again $\det(\mathbf{A}) = 0$, but now $\omega_{3,4} = (\Delta\omega/2) \pm \Omega$, $\Omega^2 = (\Delta\omega/2)^2 + \overline{G}(1,1) + \overline{G}(2,2)$, so that the atoms have different Rabi frequencies. Introducing the new variable C_3 according to

$$\dot{C}_3 - i\Delta\omega C_3 = -i\left(\chi_1 C_1 + \chi_2 C_2\right), \tag{17}$$

the system (10) can be reduced to a simple form

$$\dot{C}_1 = -i\chi_1 C_3, \, \dot{C}_2 = -i\chi_2 C_3. \tag{18}$$

From direct calculations we have found that such a case is fulfilled for a spherical structure with a 7-layered system (microsphere coated with 5 alternating $\lambda/4$ layers), see Fig.2 with $R_2 = 1.97\mu m$, $R_1 = 1\mu m$. Two atoms having tangentially oriented dipoles $\mathbf{d} \perp \widehat{\mathbf{r}}$ are at positions $a_1 = 0.9\mu m$ and $a_2 = 1.1\mu m$ correspondingly. In Fig.2(d) we show the radial distribution of the Green tensor component $\mathrm{Im}(\mathbf{G}_{\varphi\varphi})$ for the first atom in the microsphere. We have found that for this structure $\chi_1 = 1.72$, $\chi_2 = 0.608$ and $\chi_2/\chi_1 = 0.35$. As $\chi_1 > \chi_2$ we can see that in this case the coupling constant is larger for the first atom or, in other words, the first atom is placed in a stronger field mode of the coated microsphere.

III. EFFECTIVE HAMILTONIAN DYNAMICS

In the lossless case the Hamiltonian corresponding to the simplified situation in the single-mode regime described by Eq.(6) can be represented in the following form

$$H = \omega_f a^\dagger a + \omega_{at}\left(s_{z1} + s_{z2}\right) + \chi_1\left(as_{+1} + h.c.\right) + \chi_2\left(as_{+2} + h.c.\right), \tag{19}$$

where the effective coupling constants $\chi_i = \overline{G}(i,i)^{1/2}$ depend on the positions of atoms inside the microsphere. Because the coupling constant is larger for the atom placed in the region of a stronger field mode, the configuration is not symmetrical with respect to the permutation of the atoms. In this case the state vector is given by

$$\left|\Psi(t)\right\rangle = C_1(t)\left|0\right\rangle\left|e_1 g_2\right\rangle + C_2(t)\left|0\right\rangle\left|g_1 e_2\right\rangle + C_3(t)\left|1\right\rangle\left|g_1 g_2\right\rangle, \tag{20}$$

where $C_{1,2}(t)$ are solutions of the Eqs.(17),(18):

$$C_1(t) = -\chi_1 r(t) + \lambda,\ C_2(t) = -\chi_2 r(t) + 1 - \lambda,\ C_3(t) = -i\left(\chi_\lambda/\Omega\right)\exp(i\Delta\omega t/2)\sin(\Omega t), \tag{21}$$

with

$$r(t) = \left(\chi_\lambda/\chi^2\right)\{\exp(i\Delta\omega t/2)\left[i\frac{\Delta\omega}{2\Omega}\sin(\Omega t) - \cos(\Omega t)\right] + 1\}, \tag{22}$$
$$\chi_\lambda = \chi_1\lambda + \chi_2(1-\lambda),\ \chi^2 = \chi_1^2 + \chi_2^2,$$

and the initial conditions $C_1(0) = \lambda$, $C_2(0) = 1-\lambda$, $(\lambda = 1, 0)$ are considered (for the $\lambda = 0$ case the system evolves from the initial state $\left|g_1 e_2\right\rangle$) . In particular, the average photon number can be easily calculated using the solution (21):

$$\langle n\rangle = \left|C_3(t)\right|^2 = \left(\chi_\lambda/\Omega\right)^2\sin^2(\Omega t).$$

The reduced atomic density matrix for the state (20) has the form

$$\rho^a = Tr_f\{\left|\Psi\right\rangle\left\langle\Psi\right|\} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \left|C_1\right|^2 & \left|C_1C_2\right| & 0 \\ 0 & \left|C_1C_2\right| & \left|C_2\right|^2 & 0 \\ 0 & 0 & 0 & \left|C_3\right|^2 \end{bmatrix}. \tag{23}$$

In the frame of the standard approach [30] we obtain from (23) the concurrence $C(t)$ for two atom system as

$$C(t) = 2\left|C_1 C_2\right|. \tag{24}$$

In Fig.3 we show the dynamics of tangle C^2 (see Eq.(24)) for the cases $\lambda = 0$ (Fig.3(a)), and $\lambda = 1$ (Fig.3(b)). For this configuration the condition $\chi_1 > \chi_2$ is fulfilled, in Fig.3(a) the first atom being in the ground state is placed in a larger field strength. In this case the tangle C^2 has the form of well resolved periodical plateau. The inverse situation is shown in Fig.3(b). In Fig.3 the dashed line shows the average photon number $\langle n\rangle$. One can observe that the amplitude of rapid oscillations in the upper part of a long periodical tangle evolution (plateau) C^2 is essentially less in the case when an unexcited atom is placed in the region of stronger field (Fig.3(a)). It is easy to see, that such oscillations have the Rabi frequency and are related to the instantaneous average number of photons (dashed line in Fig.3) stored in the field.

It is worth noting that in the case when $\overline{G}(1,1)\cdot\overline{G}(2,2) \neq \overline{G}(1,2)^2$ the general formulae (10) should be used to study the dynamics of concurrence C. The evolution of the concurrence for the general situation is shown in Fig.4 (solid line). The particular situation described by the simplified model (21) is presented in the same Fig.4 as the dashed line. From Fig.4 one can observe that if (16) is fulfilled the amplitude of the fast oscillations is much less than in the general case, and therefore can be regarded as the optimal dynamics.

For the far detuned case $|\Delta\omega| >> \chi$ one can easily obtain from (21) two well separated frequency components of $C_{1,2}(t)$:the high frequency component $\exp(i\Delta\omega t)$ and the low frequency component $\exp(ig^2 t/\Delta\omega)$. The latter leads to a formation of well recognized plateaus, which do not exist in the resonant case. Further, we will use the initial conditions corresponding to $\lambda = 1$, so that $\chi_\lambda = \chi_1$. The concurrence (24) for such a solution can be easily analyzed in parameter space χ_1, χ_2 for the case $\Delta\omega = 0$. In this case concurrence (24) is explicitly asymmetrical with respect to the atomic permutation ($\chi_1 \rightleftarrows \chi_2$) and has the form

$$C(\chi_1,\chi_2) = 2\frac{\chi_1\chi_2}{\Omega^2}\left|1 - \frac{\chi_1^2}{\Omega^2}\left[1-\cos(\Omega t)\right]\right| \cdot \left[1-\cos(\Omega t)\right]. \tag{25}$$

It is easy to see from (25) that the surface $C(\chi_1,\chi_2)$ is separated by circles (with radii $\left(\chi_1^2+\chi_2^2\right)^{1/2} = 2k\pi/t,\ k = 0,1,2..$) on which $C(\chi_1,\chi_2) = 0$ (atoms are disentangled). The

FIG. 3: Dynamics of the two-atom tangle C^2 vs $\tau = \omega_{at}t$ for $\chi_1 = 0.254$ and $\chi_2 = 0.151$ in a lossless case for (a) initial state $|g_1 e_2\rangle$; (b) initial state $|e_1 g_2\rangle$. Due to large detuning $\Delta\omega/\omega_{at} \simeq 0.5$ the amplitude of mean photon number $\langle n \rangle$ oscillations is less in (a) case. One can see the well-recognized plateaus of tangle in case (a). In case (b) the plateaus is adding with strong oscillations due to large $\langle n \rangle$.

detailed structure of the concurrence can be better understood rewriting Eq.(25) in the form

$$C(k, a) = 2\frac{ak}{1+k^2}\left|1 - \frac{a}{1+k^2}\right|, \tag{26}$$

where $k = \chi_2/\chi_1$ and the quantity $a = 1 - \cos(\Omega t)$ is in the range $0 \leq a \leq 2$. At a fixed value of a the concurrence $C(k, a)$ assumes maximal values $C(k, a) \leq 1$ at $k_{1,2} = 2^{-1}\left[6a \pm 2\left(9a^2 - 4a + 4\right)^{1/2}\right]^{1/2}$, and $C(k, a) = 1$ only at $a = 2$ when $k^{\pm} = \sqrt{2} \pm 1 = 1/k^{\mp}$. Note that k_2 exists for $a \geq 1$. In general $1 \leq k_1 \leq k^+ \approx 2.41$ and $0 \leq k_2 \leq k^- \approx 0.41$. This means that in the resonant case, $\Delta\omega = 0$, the two-atom system can be maximally entangled if $\chi_2/\chi_1 = k^{\pm}$, i.e. when the atoms have different field-atom coupling constants, $\chi_2 \neq \chi_1$. Nevertheless in the general case, when $\Delta\omega \neq 0$, the structure of the concurrence $C(\chi_1, \chi_2)$ is more complicated.

The structure $C(\chi_1, \chi_2)$ was calculated for $\omega_{at}t = 27$, $\lambda = 0$ (initially excited atom is

FIG. 4: Tangle C^2 vs $\tau = \omega_{at}t$ in general case for parameters $\Delta\omega/\omega_{at} = 0.75$, and (a) $G(1,1) = 0.01$, $G(1,2) = G(2,1) = 0.012$ and $G(2,2) = 0.04$. Dash line shows tangle for $G(1,1) = 0.01$, $G(1,2) = G(2,1) = 0.02$ and $G(2,2) = 0.04$ when Eq.(16) is valid. In case (b) $G(1,1) \leftrightarrows G(2,2)$.

placed in a smaller field) and $\Delta\omega/\omega_{at} = 0.5$. We observe that the surface $C(\chi_1, \chi_2)$ is rather asymmetrical with respect to the line $\chi_1 = \chi_2$. In the course of evolution for fixed χ_1, χ_2 the maximal values of $C(\chi_1, \chi_2)$ move out from the origin of coordinates. Obviously on the edges where $\chi_1, \chi_2 = 0$, the concurrence vanishes, $C(\chi_1, \chi_2) = 0$. In the vicinity of maxima the concurrence $C(\chi_1, \chi_2)$ is highly asymmetric. For $\chi_1 > \chi_2$ the maxima $C(\chi_1, \chi_2)$ in the left side are smoother and the hills are more pronounced. This means that the system remains in the region of strong entanglement for long periods if $\chi_1 > \chi_2$. However in general, the details of the surface $C(\chi_1, \chi_2)$ essentially depend on $\Delta\omega$ and the form of the Green's function.

IV. NUMERICAL STUDY

It is worth noting, that in a real microsphere the field dissipation is caused by material losses and the radiation into surrounding space leads to line broadening (bandwidth). The

FIG. 5: The same as in Fig.3 but for loss case ($\gamma = 2 \cdot 10^{-2}$). To see the details of long-time dynamics we calculate C^2 up to $\tau_{\max} = 1000$.

analytical calculation of such a broadening requires an extensive knowledge of the microscopical local field, which in a multilayered microsphere case is itself a quite difficult problem. To estimate the influence of the dissipation on the concurrence dynamics we will use the following simplified approach. Although we do not know the exact frequency dependence of the dissipative part on the refractive indices of the materials n_i in a microsphere, it is possible to calculate the spectral width of the Green function peak. Thus, we can estimate the effect of the field's dissipation using the master equation technique in the framework of the Lingblad approach. In particular, the dissipation coefficients are calculated from the bandwidth of the Green function peak (see Fig.2). Such a semi-analytical approach allows us to simulate numerically not only the evolution of the concurrence in a lossy environment, but also the dynamics of the average photon number. In this approach we replace the exact Hamiltonian (1) by the simplified Hamiltonian (19) and numerically solve the following master equation for the joint atom-field density operator ρ in a dissipative cavity at zero temperature:

$$\frac{d\rho}{d\tau} = -i[H, \rho] + L_1 \rho, \tag{27}$$

$$L_1 \rho = \gamma_1 \left(2a\rho a^+ - a^+ a \rho - \rho a^+ a \right), \tag{28}$$

where H is given by (19), and in (27) we neglect the atomic spontaneous emission in the Rabi period time scale. Also we have used the detuning $\Delta\omega/\omega_{at} = 0.5$, and $\chi_1 = 0.254$, $\chi_2 = 0.151$, $\chi_2/\chi_1 = 0.594$. In Fig.5 the dynamics of the two-atom tangle C^2 for the lossy case is shown. It is clear from Fig.5 that the plateaus of concurrence survive even in the presence of dissipation, although their amplitude is obviously lower than in the lossless case.

V. CONCLUSION

In conclusion, we have studied the dynamics of entanglement of spatially separated two-level atoms interacting with a radially nonuniform cavity field mode in a dielectric microsphere coated with an alternating stack. We found that due to the field inhomogeneity the atoms can be maximally entangled even in the resonant case. We have found that entanglement essentially depends on the atomic positions (asymmetrical entanglement) and also on the detuning between atoms and the field mode frequencies. The entanglement is considerably more stable with duration much longer than the period of Rabi oscillations (robust entanglement) when the unexcited atom is placed in a stronger field, while the detuning increases the duration of the entanglement period. The dissipation reduces the amplitude of the entanglement, however practically does not change the width of the zones of large entanglement.

VI. ACKNOWLEDGEMENTS

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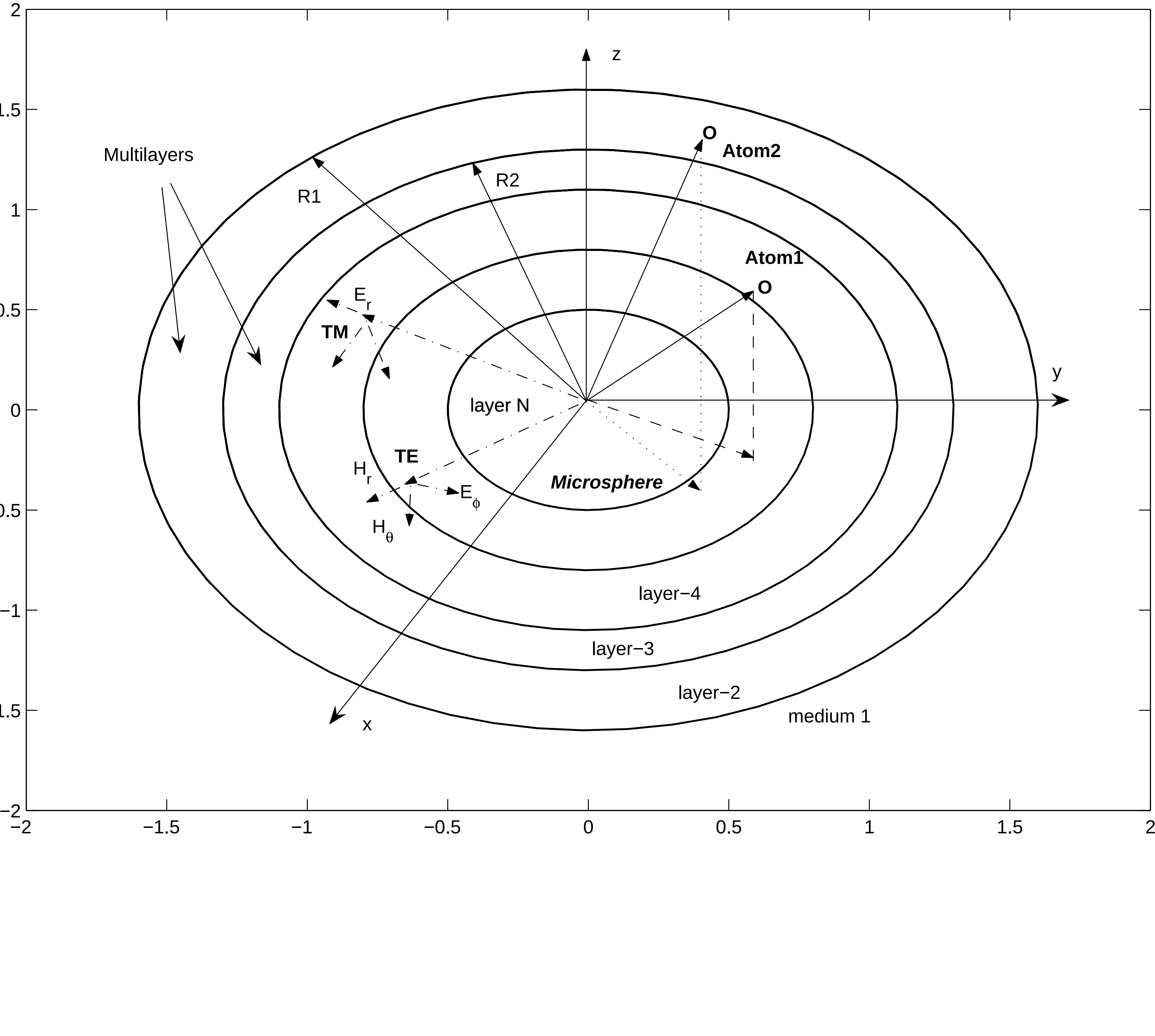

z
y
x
Multilayers
R1
R2
O
Atom2
Atom1
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layer N
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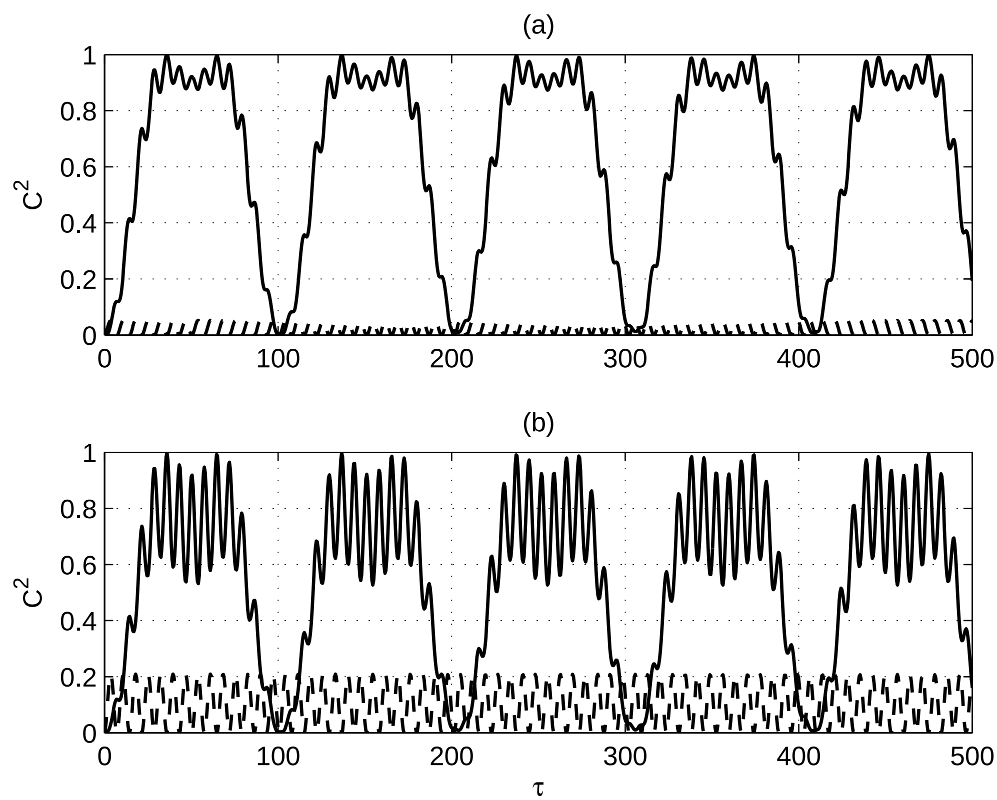
layer−2
medium 1



(a)
1
0.8
0.6
0.4
0.2
0
C^2
0
100
200
300
400
500
(b)
1
0.8
0.6
0.4
0.2
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C^2
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200
300
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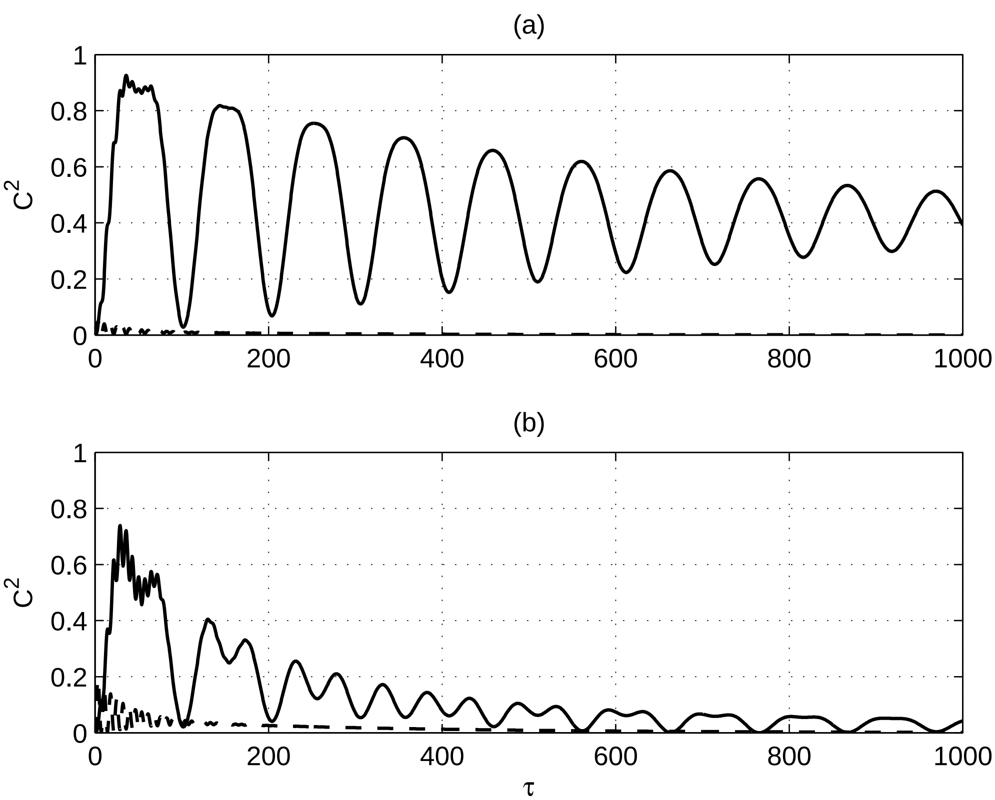
500
τ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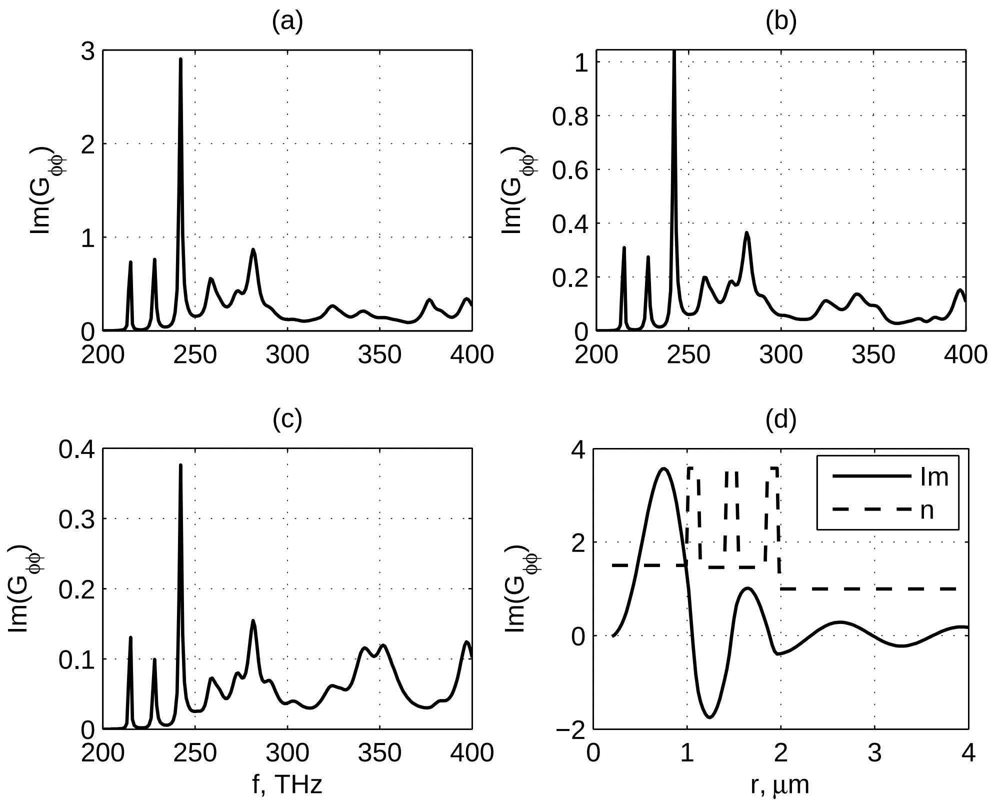
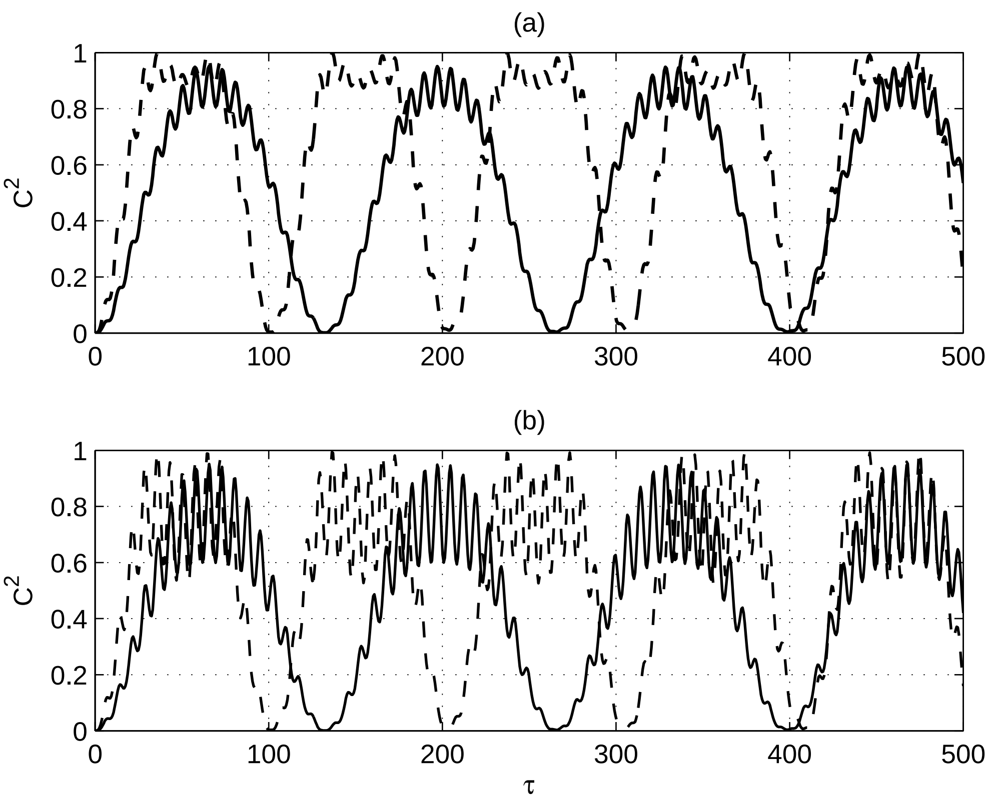


(a)
(b)
(c)
(d)
Im(G_φφ)
f, THz
r, μm
Im
n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